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MTTF, FAILRATE, RELIABILITY AND LIFE TESTING

by Bob Seymour

At Burr-Brown, we characterize and qualify the reliability of our devices through high temperature life testing. The results of this testing are quantified with such values as MTTF and failure rate. This information can be very valuable when used for comparative purposes or applied to reliability calculations. However, this information loses its worth if it is not precisely understood and appropriately employed. It is the intent of this application note to bring together, in a concise format, the definitions, ideas, and justifications behind these reliability concepts in order to provide the background details necessary for full and correct utilization of our life testing results.

SOME PRELIMINARY DEFINITIONS

Reliability is "the probability that a part will last at least a specified time under specified experimental conditions"(1).

MTTF is the mean time to the first failure under specified experimental conditions. It is calculated by dividing the total number of device • hours by the number of failures. It is important to note, at this time, that the dimensions of MTTF are not hours per failure, but rather, device • hours per failure. If each part has a 0.1% chance of failure before 1 hour then 10 parts have a 1% chance experiencing a failure by that time. The MTTF will be the same in both cases. 1 failure in 10 hours on 1 part or 1 failure in 1 hour on 10 parts both produce an MTTF of 10 device • hours.

Failure rate is the conditional probability that a device will fail per unit of time. The conditional probability is the probability that a device will fail during a certain interval given that it survived at the start of the interval.(5) When failure rate is used to describe the frequency with which failures are expected to occur, the time units are typically device • hours.

FITS is simply failure rate scaled from failures per device • hour to failures per billion device • hours.

ON TO THE DETAILS

In the definition section MTTF is defined as the average time, in device • hours, per failure observed under specific experimental conditions such as a life test. Here at Burr-Brown we use a slightly modified formula for MTTF. We calculate 2 times the total device • hours, T_{dh}, divided by the upper 60% confidence limit of a chi-square distribution with 2 times the observed number of failures + 2 degrees of freedom, $X^2(2f + 2)$. Our formula is

$$\text{MTTF} = \frac{2T_{dh}}{\chi^2(2f+2)}$$

Since both time and failures are doubled, these definitions are roughly equivalent. Some explanation is in order.

If multiple life tests are run on the same type of device, it is unlikely that all tests will have the same number of failures for the same number of device • hours. Rather there will be a distribution of failures. The minimum value must be 0 for no failures. The maximum value could correspond to 100% failures, but we can presume that we are running enough parts that this will not happen. Rather the distribution will taper off as the number of failures increases. Somewhere in between there will be a concentration of failures.

The chi-square calculation provides us with a tool for adjusting the actual number of failures from a limited life test to make it more accurately reflect what we might expect from the population as a whole. For example, applying a confidence level of 60% to a chi-square distribution with 8 degrees of freedom will return a value into the denominator of the MTTF calculation which is greater than or equal to 60% of the values in a chi-square distribution with a mean of 8.

One intuitive interpretation of the chi-square calculation is that the calculated value represents, roughly, a number of failures which will be greater than 60% of the failures we might get during multiple life tests. The upper 60% level is selected because it represents an approximately average estimate for MTTF and because it is widely accepted among semiconductor manufacturers and users. This method of estimating MTTF does not prevent further reliability calculations from being made at more conservative levels.

One more point remains to be explained regarding this calculation. Why do we use 2 (# failures) +2? The technical explanation for this is given later in this paper. Briefly, the factor of 2 is necessary to achieve theoretical validity of the X^2 distribution. Given the factor of 2, it can be seen that we are merely adding 1 failure to the actual number of failures. The added failure appears in the calculation as if a failure occurred at the end of the test. This assures that the test terminates with a failure, also a theoretical requirement, as well as allows calculation of MTTF even if no failures were observed.

The MTTF value by itself really only serves for comparison purposes. Many more factors need to be considered before predictive statements regarding the longevity of our components can be made. The statistical concepts of reliability and failrate allow us to make such predictions. I will present here, with justification yet to come, the statistical formulas which quantify these concepts.

Let reliability be represented by R(t) and failure rate by Z(t).

Then $$R(t) = e^{-\alpha t^{\beta}}$$

and $$Z(t) = \alpha\beta\, t^{\beta-1}$$

Remember, reliability is the probability that a part will function at least a specified time. Failure rate describes the frequency with which failures can be expected to occur. By examining failure rate we can make important statements about the life cycle of the product.

The life cycle of a part can be thought of as having three distinct periods: infant mortality, useful life, and wear-out. These three periods are characterized mathematically by a decreasing failure rate, a constant failure rate, and an increasing failure rate. This theory is the basis of the ubiquitously discussed "bathtub curve".

The listed formulas can model all three of these phases by appropriate selection of α and β. β affects the shape of the failure rate and reliability distributions. When $\beta < 1$ Z(t) becomes a decreasing function. $\beta = 1$ provides a constant failure rate. An increasing failure rate can be modeled with $\beta > 1$. Therefore, β can be selected to accurately model the shape of an empirically known failure rate (or of the original probability density function of T which defines the failure rate). The constant α provides the scaling factor.

Given good design, debugging, and thorough testing of product the infant mortality period of a part's life should be past by the time the parts are shipped. This allows us to make the assumption that most field failures occur during the useful life phase, and result, not from a systematic defect, but rather from random causes which have a constant failure rate. The constant failure rate presumption results in $\beta = 1$. Thus

$$Z(t) = \alpha$$

The concept of a constant failure rate says that failures can be expected to occur at equal intervals of time. Under these conditions, the mean time to the first failure, the mean time between failures, and the average life time are all equal. Thus, the failure rate in failures per device • hour, is simply the reciprocal of the number of device • hours per failure. That is

$$Z(t) = \alpha \approx 1/\text{MTTF}$$

during constant failure rate conditions.

Note that MTTF is always the number of device • hours per failure but neither failure rate nor α is always 1/MTTF.

FORMAL DERIVATIONS AND JUSTIFICATIONS

OK, it's time for some real details. Virtually all of the information on the Weibull distribution comes from *"Probability and Statistics for Engineers and Scientists"* by Ronald E. Walpole and Raymond H. Myers, copyright 1985, Macmillan Publishing Company. Much of the information in the section on MTTF is extrapolated from the lectures of Dr. Duane Dietrich, professor of Systems and Industrial Engineering at the University of Arizona. My apologies to Dr. Dietrich for any distortions.

Let's start by hypothetically running a huge life test long enough to drive all devices to failure, recording time-to-failure for each part, generating histograms, calculating MTTF, etc. A histogram of time-to-failure would be useful. Its shape is unknown and unimportant at this time. Given the hypothetical nature of the experiment, we can presume that the distribution is representative of the whole population.

From this distribution, we can describe reliability as

$$R(t) = P\ (T > t)$$

where T represents time-to-failure and t represents time. Note that this is merely an exact restatement of the verbal definition already presented.

Another useful function which can be derived from the time-to-failure histogram will represent the cumulative probability of failure at any time t. Let F(t) represent this function. Then

$$F\ (t) = P\ (T < t)$$

or

$$F\ (t) = 1 - R\ (t)$$

Now we are positioned to examine failure rate. Failure rate is the conditional probability that a device will fail during a certain interval, given that it survived to the start of that interval, per unit of time. Let Z(t) represent failure rate. Then

$$Z(t) = \lim_{\delta t \to 0} \frac{F(t+\delta t) - F(t)}{R(t)\delta t}$$

Now note that

$$\lim_{\delta t \to 0} \frac{F(t+\delta t) - F(t)}{\delta t}$$

is the derivative of F(t). Also F(t) = 1 – R(t). Therefore, dF(t)/dt = –dR(t)/dt. Thus

$$Z(t) = \frac{dF(t)}{R(t)dt}$$

$$= \frac{-dR(t)}{R(t)dt}$$

$$= \frac{-d\left[\ell n\ R(t)\right]}{dt}$$

Integrating both sides results in

$$\ell n\ R(t) = -\int Z(t)dt + \ell n\ c$$

giving

$$R(t) = c\ e^{-\int Z(t)dt}$$

as the relationship between reliability and failure rate based only on the original definitions. The constant, c, must satisfy the initial condition that all parts are assumed to be functional at time t = 0 or R(0) = 1.

ENTER WALODDI WEIBULL

The statistical distribution introduced by Waloddi Weibull in 1939 provides the mechanism to make our reliability function usable. For x > 0 the distribution is given by

$$f(x) = \alpha\beta x^{\beta-1}e^{-ax^{\beta}}$$

where $\alpha > 0$ and $\beta > 0$.

Let us presume that the original probability density distribution of T (time-to-failure) is describable using the Weibull distribution. Then

$$R(t) = 1 - F(t) = 1 - \int_o^t \alpha\beta x^{\beta-1}e^{-\alpha x^{\beta}}dx$$

$$R(t) = 1 + \int_o^t de^{-\alpha x^{\beta}}$$

$$R(t) = e^{-\alpha t^{\beta}} \text{ (see appendix A)}$$

and

$$Z(t) = \frac{-dR(t)}{R(t)dt} = \frac{\alpha\beta t^{\beta-1}e^{-\alpha t^{\beta}}}{e^{-\alpha t^{\beta}}}$$

$$Z(t) = \alpha\beta\, t^{\beta-1}$$

Thus, the Weibull distribution provides usable mathematical descriptions of reliability and failure rate:

$$R(t) = e^{-\alpha t^{\beta}}$$

$$Z(t) = \alpha\beta\, t^{\beta-1}$$

But do these agree with our formulas derives strictly without presuming the Weibull distribution? This definition of Z (t) can be entered into our previous derivation to justify our assumption.

$$R(t) = c\, e^{-\int Z(t)dt}$$

$$R(t) = c\, e^{-\int \alpha\beta t^{\beta-1}dt}$$

$$R(t) = c\, e^{-\alpha t\beta}$$

For R (0) = 1 then c = 1 and

$$R(t) = e^{-\alpha t\beta}$$

as before. Thus, the Weibull distribution fits our original definitions, provides a solution to the original equations, and results in useful formulas for reliability and failure rate.

More on constant failrate and MTTF.

We presume constant failrate conditions during our life test evaluations. It is particularly important to understand this condition well. What are constant failrate conditions? How do they affect the Weibull equations? And what, exactly, is MTTF?

During the useful life period of our parts, there are no systematic defects or problems causing a high early failure rate nor an increasing rate of failure associated with aging. Failures during this period result from random causes. The probability of a part failing for a random defect or stress does not change as the part ages. The failure rate, the conditional probability that a part will fail at a specific time, T, given that it has survived to that time, is constant.

From the Weibull distribution, the general equation for failure rate is given by

$$Z(t) = \alpha\beta t^{\beta-1}$$

Given that Z(t) must equal a constant then b must equal 1 to drive the time variable t to unity. Thus, under constant failure rate conditions, the equations for failure rate, reliability and the Weibull distribution, become, respectively

$$Z(t) = \alpha$$

$$R(t) = e^{-\alpha t}$$

and

$$f(t) = \alpha\, e^{-\alpha t}$$

The function f(t) is the time-to-failure probability density function. It gives the probability that a part will fail at any given time t. The mean, or expected value, of f(t) is the average time-to-failure. This mean value is equal to $1/\alpha$. The problem is that we do no know the true value of $1/\alpha$. This value must be estimated from experimental data.

An estimator for $1/\alpha$ can be derived using the maximum likelihood method with the function f(t). Suppose we run a life test starting with N parts and experience r failures. The joint probability density function describing the life test results is given by the product of the probabilities that each failure occurred when it did. Referring to this p.d.f. as L (α, t) then

$$L(\alpha, t) = \alpha^r\, e^{-\alpha\sum_{i=1}^{r} t_i}$$

Implicit in this derivation is that the life test is terminated at the r^{th} failure and the dimension of t is device • hours. Our method of evaluating MTTF involves adding 1 failure to the observed failures. This assures the requirement for termination on the r^{th} failure is satisfied as well as allows calculation of MTTF even if no actual failures occur. The dimensioning of t as device • hours accounts for the test time of those parts that did not fail.

To find an appropriate estimator for $1/\alpha$ by the maximum likelihood method, we find the value of α which maximizes the function L (α, t). We are, in effect, finding the value of

α which maximizes the probability of observing what was actually observed. This is accomplished by taking the partial derivative of the $\log_e$ of L (α, t), setting it equal to zero, and solving for α:

$$\ell n\,(L(\alpha,t)) = r\ell n\,(\alpha) - \alpha\sum_{i=1}^{r} t_i$$

$$\frac{\partial \ell n\,(L(\alpha,t))}{\partial\alpha} = \frac{r}{\alpha} - \sum_{i=1}^{r} t_i$$

$$\frac{r}{\tilde{\alpha}} - \sum_{i=1}^{r} t_i = 0$$

using $\tilde{\alpha}$ to indicate the approximation,

$$\frac{1}{\tilde{\alpha}} = \frac{\sum_{i=1}^{r} t_i}{r}$$

This equation shows that $1/\alpha$ can be estimated by dividing the accumulated test time for all of the tested devices by the total number of failures. This agrees with the original definition of MTTF.

Understanding the chi-square, X^2, confidence interval calculation requires recognition that given the random variable for time-to-failure, T, has distribution

$$f(t) = \alpha e^{-\alpha t}$$

then the random variable V described by

$$V = 2\alpha\sum_{i=1}^{r} t_i$$

is distributed X^2 with 2r degrees of freedom, X^2(2r). Therefore, for a specified confidence level ç

$$2\alpha\sum_{i=1}^{r} t_i > X^2(2r,\zeta)$$

and the upper confidence limit for MTTF becomes

$$\frac{1}{\alpha} < \frac{2\sum_{i=1}^{r} t_i}{\chi^2(2r,\zeta)} = \text{MTTF}$$

which, with r equal to the number of observed failures + 1, is the actual formula we use for MTTF.

To justify the X^2 distribution of the random variable V used above, apply the transformation of variable

$$V' = 2\alpha T, \frac{dT}{dV'} = \frac{1}{2\alpha}$$

to f(t) which results in

$$f(v) = \alpha e^{-\alpha\frac{V}{2\alpha}}\,\frac{1}{2\alpha} = \frac{1}{2}e^{-\frac{V}{2}}$$

which is distributed X^2 with 2 degrees of freedom, $X^{2(2)}$ (see Appendix B).

Now note that

$$2\alpha\sum_{i=1}^{r} t_i = 2\alpha t_1 + 2\alpha t_2 + \ldots + 2\alpha t_r$$

As shown, each factor of the above sum is distributed $X^{2(2)}$. Therefore, by the reproductive property of X^2 the summation is also distributed X^2 with r times 2 degrees of freedom, X^2 (2r) as stated.

AND FINALLY

The concepts of MTTF, failure rate and reliability have been defined, discussed and justified. In general, the time units of device • hours have been used. With this dimension, failure rate can be interpreted as the frequency with which failures can be expected to occur. This description works well with the experimental estimation of the unknown parameters and provides an intuitive perspective. However, reliability estimation is, in essence, a probabilistic science and the Weibull equations are, in essence, probability equations. As a probability equation, failure rate becomes the probability of failure per hour, not per device • hour. The reader is encouraged to give this distinction some thought.

We perform our life testing at elevated temperatures in order to accelerate failure mechanisms which might result in device failure. Our reliability reports generally supply MTTF estimates scaled over a range of temperatures appropriate to application environments. The Arehnius equation with an activation energy selected to represent typical failure mechanisms is employed to generate the tables.

Here at Burr-Brown we use a spreadsheet program to calculate and record the results of life tests. Constant failure rate is presumed. This presumption should always be verified. It may not be unreasonable to interpret MTTF as the mean time to the first failure even if the failure rate is not constant. However, failure rate and reliability predictions based on that MTTF will be wrong.

APPENDIX A

$$\begin{aligned} d\exp(-\alpha x^\beta)\,/\,dx &= \exp(-\alpha x^\beta)\,d(-\alpha x^\beta)\,/\,dx \\ &= \exp(-\alpha x^\beta)\,(-\alpha\beta)\,(x^{\beta-1}) \\ &= -\alpha\beta x^{\beta-1}\exp(-\alpha x^\beta) \end{aligned}$$

therefore replacing

$$\begin{aligned} -\int_0^t \alpha\beta x^{\beta-1}\exp(-\alpha x^\beta)\,dx &= -\int_0^t d\exp(-\alpha x^\beta)\,/\,dx\,dx \\ &= -\int_0^t d\exp(-\alpha x^\beta) \end{aligned}$$

and

$$\begin{aligned} R(t) &= 1 + \int_0^t d\exp(-\alpha x^\beta) \\ &= 1 + \exp(-\alpha x^\beta)\,\Big|_0^t \\ &= 1 + \exp(-\alpha t^\beta) - 1 \end{aligned}$$

APPENDIX B

The chi-square distribution with r degrees of freedom is given by

$$f(v) = \frac{1}{2^{\frac{r}{2}}\Gamma\left(\frac{r}{2}\right)} v^{\frac{r}{2}-1} e^{-\frac{v}{2}}$$

Setting r = 2 for $X^{2(2)}$ results in

$$\frac{1}{2^{\frac{2}{2}}\Gamma\left(\frac{2}{2}\right)} v^{\frac{2}{2}-1} e^{-\frac{v}{2}} = \frac{1}{2} e^{-\frac{v}{2}}$$

as indicated.

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